DEVELOPMENT OF A CAMOUFLET CAVITY AS A RESULT OF AN EXPLOSION IN SOFT GROUND

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The problem of an explosion is considered on the basis of a model of a viscoplastic medium. It follows from the solution that the theoretical dependence of the cavity radius on the time coincides satisfactorily with the experimental data. The introduction of viscosity into the scheme of the motion leads to the appearance of the scaling effect. From an analysis of the stress field, a conclusion follows concerning the existence, even in the initial stage of motion, of intense tangential tensile stresses, which lead to the formation of a system of radial fissures, observed in the experiment.

A large amount of theoretical work has been devoted to the problem of an explosion [1-5]. The purpose of this work was to obtain and estimate the various parameters which characterize a fully contained explosion in the ground: the radius developed by the camouflet cavity, its time of development, intensity of the radiated elastic waves, etc. Obviously, within the scope of a single theoretical model it is difficult to describe the whole process from the start of detonation to the total cessation of motion. In particular, the intensive cracking of the cavity boundary, which is characteristic of a fully contained explosion, is not explained. Recently, papers have appeared in print whose authors attempt to explain this phenomenon [6, 7]. The explanation for this is founded on the basis of the well-known model of the ground, but with the appropriate choice of parameters for the condition of plasticity, far from reality [6], which makes this explanation unlikely, or has been based on semiempirical criterial estimates [7]. Nevertheless, such an instability of the gas cavity boundary is needed in a more obvious physical representation and explanation.

It is clear "a priori," that the formation of a system of radial fissures can occur by the action of intense tangential tensile stresses. In the proposed paper, such a stress field is constructed on the basis of a model of a viscoplastic medium. It should be noted that the idea of taking account of the viscous properties of the ground in the solution of dynamic problems clearly belongs to Lyakhov, having determined experimentally the viscosity of sand [8] and who used this model for solving wave problems [9].

We shall construct the solution on the basis of the most simple scheme of motion – a model with constant packing. This model, which is mathematically simple, describes well all the principal characteristics of the phenomenon. By analogy with [2], we represent the motion of the ground in the following way: At the instant of time $t=0$ and in a cavity with initial radius a_0, an instantaneous release of energy occurs, after which a shock wave is propagated through the ground. The medium behind the wave front is uncompressed and, according to the concepts assumed by us, it is viscoplastic. The problem will be solved in the case of central symmetry. The equations of motion and continuity have the form

$$\rho\left(\frac{du}{dt}+u\frac{du}{dr}\right)=\frac{d\sigma_r}{dr}+\frac{2(\sigma_r-\sigma_\theta)}{r}; \tag{1}$$

$$\frac{\partial}{\partial r}(r^2u)=0, \tag{2}$$

where u is the bulk velocity; σ_r and $\sigma_\theta=\sigma_\varphi$ are the components of the stress tensor; ρ is the density of the medium behind the shock-wave front.

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TABLE 1

	Sand	Loess	Clay
m	0,45	0,425	0,233
$k\frac{\text{kg}}{\text{cm}^2}$	0,355	0,96	1,42

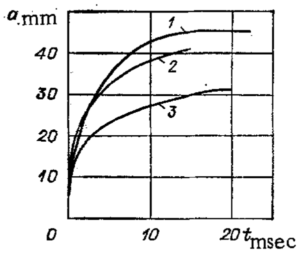

Fig. 1

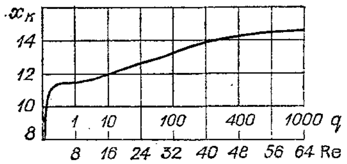

Fig. 2

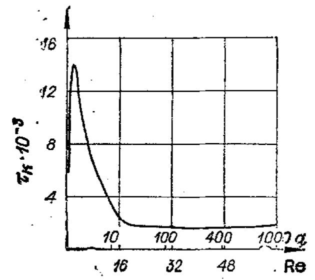

Fig. 3

In the usual form, the relation between the components of the stress tensor and the tensor of the rate of deformation for an incompressible, viscoplastic medium is written in the form [10]

$$s_{ij} = 2\left(\frac{\tau_s}{H} + \frac{\mu}{3}\right) e_{ij}, \tag{3}$$

where s_{ij} are the components of the stress deviator; e_{ij} are the components of the tensor of the rate of deformation; $H = (2e_{ij}, e_{ij})^{1/2}$ is the intensity of the rate of deformation; τ_s is the Mises yield point; μ is the coefficient of humidity. For soils, the yield point depends on the hydrostatic pressure, and the condition of plasticity is known from experiments [5]:

$$s_{ij} \cdot s_{ij} = 2(k + mp)^2, \tag{4}$$

where m and k are constants, characterizing the internal friction and the ground coupling; $p = -(\sigma_r + 2\sigma_\theta)/3$ is the hydrostatic pressure. It is proposed below to combine conditions (3) and (4) for describing the stressed state of the ground. Then, in the case of spherical symmetry, the condition of plasticity, taking account of the rate of deformation, assumes the form

$$\sigma_r - \sigma_\theta = -\kappa + m(\sigma_r + 2\sigma_\theta) + \frac{2}{3}\mu\left(\frac{\partial u}{\partial r} - \frac{u}{r}\right)$$

or, using the equation of continuity,

$$\sigma_r - \sigma_\theta = -k + m(\sigma_r + 2\sigma_\theta) - 2\mu u/r. \tag{5}$$

Solving Eqs. (1) and (2), we obtain

$$u = \lambda(t)/r^2; \tag{6}$$

$$\sigma_r = \frac{k}{3m} + \frac{\rho\dot{\lambda}}{(\alpha - 1)r} - \frac{2\rho\lambda^2}{(\alpha - 4)r^4} + \frac{2\mu\alpha\lambda}{3m(\alpha - 3)r^3} + r^{-\alpha} \cdot C(t),$$

where $\alpha = 6m/(1 + 2m)$, and the dot denotes differentiation with respect to time.

The problem is solved for the following boundary conditions: at the cavity boundary, $r = a$,

$$\sigma_r(a) = -P(a),$$

at the shock-wave front, $r = R$

$$u(R) = \xi\dot{R};$$
$$\sigma_r(R) = -\rho_0\xi\dot{R}^2 - P_*,$$

where $\xi = 1 - \rho_0/\rho$ is the "packing" of the ground; ρ_0 is the initial density; P^* is the initial pressure in the ground.

From the law of conservation of mass of matter, we obtain the relation expressing the connection between the cavity radius and the coordinate front of the shock wave,

$$R = \left[\frac{1}{\xi} - \frac{1 - \xi}{\xi}\left(\frac{a_0}{a}\right)^3\right]^{1/3} \cdot a = \varepsilon(a)\,a. \tag{7}$$

We determine the function $\lambda(t)$ at the cavity boundary,

$$\lambda(t) = a^2\dot{a}. \tag{8}$$

Substituting the expression for σ_r in the boundary conditions, and using relations (7) and (8), having eliminated C(t) we obtain the ordinary differential equations relative to the cavity radius,

$$a\frac{d^2a}{dt^2}+\left[2+\frac{(1-\xi)(\alpha-1)\varepsilon^{\alpha-4}}{\xi(\varepsilon^{\alpha-1}-1)}-\frac{2(\alpha-1)(\varepsilon^{\alpha-4}-1)}{(\alpha-4)(\varepsilon^{\alpha-1}-1)}\right]\left(\frac{da}{dt}\right)^2+$$
$$+\frac{2\mu\alpha(\alpha-1)(\varepsilon^{\alpha-3}-1)}{3m\rho(\alpha-3)(\varepsilon^{\alpha-1}-1)a}\cdot\frac{da}{dt}=\frac{\alpha-1}{\rho(\varepsilon^{\alpha-1}-1)}\left[P(a)-P_*\varepsilon^{\alpha}-\frac{k}{3m}(\varepsilon^{\alpha}-1)\right]. \quad (9)$$

The instant of time when $a=0$ should be taken for the cessation of motion.

We introduce dimensionless variables by the formulas

$$x=a/a_0,\quad \tau=t/a_0\sqrt{\xi P_0/\rho_0},$$

where P_0 is the initial pressure in the detonation products. Then in place of Eq. (9) we obtain

$$x\frac{d^2x}{d\tau^2}+\left[2+\frac{(1-\xi)(\alpha-1)\varepsilon^{\alpha-4}}{\xi(\varepsilon^{\alpha-1}-1)}-\frac{2(\alpha-1)(\varepsilon^{\alpha-4}-1)}{(\alpha-4)(\varepsilon^{\alpha-1}-1)}\right]\times$$
$$\times\left(\frac{dx}{d\tau}\right)^2+\frac{\mu}{a_0\rho_0}\sqrt{\frac{\rho_0}{P_0}}\cdot\frac{2\alpha(\alpha-1)(1-\xi)(\varepsilon^{\alpha-3}-1)}{3m(\alpha-3)(\varepsilon^{\alpha-1}-1)\sqrt{\xi}}\frac{1}{x}\frac{dx}{d\tau}=$$
$$=\frac{(\alpha-1)(1-\xi)}{(\varepsilon^{\alpha-1}-1)\xi}\left[\frac{P(x)}{P_0}-\frac{P_*}{P_0}\varepsilon^{\alpha}-\frac{k}{3mP_0}(\varepsilon^{\alpha}-1)\right]. \quad (10)$$

The dimensionless combination $a_0\rho_0/\mu\sqrt{P_0/\rho_0}$, which can be identified in sense with the Reynolds number (well known in viscous liquid theory, and which we shall subsequently use) is factored out in the coefficient for the third term on the left-hand side of Eq. (10).

Unfortunately, even by neglecting the initial stage of motion, i.e., when $\varepsilon\simeq\xi^{-1/3}$, it is not possible to obtain a solution in closed form. Equation (10) has been integrated by means of a computer. The function $P(a)$ was determined in the following way:

$$P(a)=\begin{cases}P_0(a/a_0)^{-3\gamma_1}, & \text{if}\quad a_*\geqslant a\geqslant a_0\\ P_0(a_*/a_0)^{-3\gamma_1}(a/a_*)^{-3\gamma_2}, & \text{if}\quad a>a_*,\end{cases}$$

where $P_0=8\cdot10^{10}$ dyn/cm^2; $\gamma_1=3$; $\gamma_2=1.27$ and $a_*=1.53\ a_0$. The initial density of the ground ρ_0 in the calculations was assumed equal to 1.6 g/cm^3 and the viscosity of the ground, $\mu=3\cdot10^5$ p, which corresponds to sand [8]. The ground packing was varied within the limits from 0.025 to 0.1. The parameters characterizing the internal friction and the ground coupling are chosen on the basis of experimental data [11] (Table 1).

An example of the numerical calculation is given in Fig. 1, where the theoretical function $a(t)$ is plotted for sand, with $\eta=0.05$; $P_*=10^6$ dyn/cm^2, corresponding to the explosion of a charge with a weight of 1 g (curve 3) and, for comparison, the experimental curves according to data from [12, 13] are plotted. Qualitatively, the behavior of the theoretical function calculated by us coincides well with the experimental data. The difference in the determination of the final cavity radius can be attributed to imperfection of the theoretical model, i.e., we must formulate more precisely the equation of state of the ground, $p(\rho)$, taking into account variable packing, and we must solve the even more complex problem. But it should be emphasized that even in this simplified formulation the introduction of viscosity into the scheme of motion with the actually acceptable experimental parameters defining the internal friction, the coupling, ground packing, the equation of state of the detonation products, and the initial pressure P_* gives good agreement with the experimental data.

The dependence of the final cavity radius x_k and the time of development τ_k on the magnitude of the packing ξ can be judged from Table 2, where the data shown corresponds to an explosion in sand with $P_*=0$ and $a_0=5.36$ cm. It can be seen from the table that the magnitude of the final radius in this case is changed insignificantly; the time of development of the cavity is changed by much more, by a factor 4.

The law of motion of the camouflet cavity can be determined as a function of a set of dimensionless parameters

$$a=a_0\cdot f(\tau,\ \mathrm{Re},\ P_*/P_0,\ k/P_0,\ m,\ \xi); \quad (11)$$

here $\mathrm{Re}=a_0\sqrt{\rho_0 P_0}/\mu$.

TABLE 2

ξ	0,1	0,075	0,05	0,025
a_k	12,86	12,21	11,4	10,23
τ_k	$0{,}1671\cdot10^5$	$0{,}1282\cdot10^5$	$0{,}858\cdot10^4$	$0{,}4174\cdot10^4$

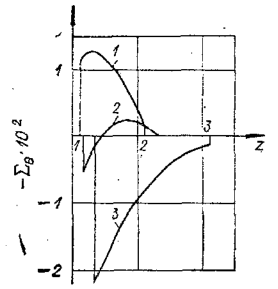

Fig. 4

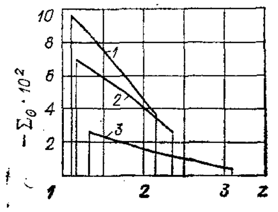

Fig. 5

It follows from Eq. (11) that camouflets produced in ground with a defined viscosity by explosions of charges with a different weight will not be geometrically similar.

The effect of the scaling effect on the final radius of the camouflet cavity can be estimated by Fig. 2, when the dependence of x_k on the Re number is plotted for sand, with $\xi = 0.05$ and $P_* = 0$. With increase of the scale of the explosion $Re \to \infty$, the function $x_k(Re)$ tends asymptotically to the value $x_k = 16.15$, i.e., at large scales the geometric similarity in the normal sense is maintained. The region of change of Re from 2 to 38 is of the greatest interest (which corresponds to an explosive charge of $\simeq 30$ g to $\simeq 200$ kg), in which the geometric similarity was verified directly by experiment. The final radius varies in this case $11.12a_0$ to $13.7a_0$, approximately by 19-23%. As the surface of the camouflet in loess, loam, or clay is perforated by numerous radial fissures and measurements of the final cavity dimensions are carried out with a finite error of $\sim 10\%$, it is difficult to detect any difference from geometric similarity.

The second most important parameter of motion (the time of final development of the cavity) varies in this case much more significantly (by a factor of several times). The function $\tau_k(Re)$, reflecting the dependence of the final time of development of the camouflet on the scaling, for values of $\xi = 0.05$ and $P_* = 0$, is plotted in Fig. 3. Here, just as in Fig. 2, the charge sizes q in kilograms, equivalent to a given value of Re, are plotted along the abcissa. Experimentally, it is important to find the difference from the geometric similarity with respect to this parameter (time of development of the camouflet), but unfortunately, we do not yet have these data available. By evaluating the behavior of the curves in Figs. 2 and 3, we arrive at the following conclusion: the effect of viscosity becomes of little importance when $Re \simeq 20$ (which is equivalent to an explosive charge of ~ 30 kg). Above a value of $Re = 20$, contained explosions in sand can be assumed, with good accuracy, to be geometrically similar.

In order to analyze the stressed state, we determine σ_r. From relation (6) and the boundary condition at the surface of the cavity we have

$$\sigma_r = \frac{\rho}{\alpha-1} a\ddot{a}\left[\frac{a}{r} - \left(\frac{a}{r}\right)^\alpha\right] + \left\{\frac{2\rho}{\alpha-1}\left[\frac{a}{r} - \left(\frac{a}{r}\right)^\alpha\right] - \frac{2\rho}{\alpha-4}\left[\left(\frac{a}{r}\right)^4 - \left(\frac{a}{r}\right)^\alpha\right]\right\}\dot{a}^2 + \frac{2\mu\alpha}{3m(\alpha-3)}\left[\left(\frac{a}{r}\right)^3 - \left(\frac{a}{r}\right)^\alpha\right]\frac{\dot{a}}{a} + \frac{k}{3m}\left[1 - \left(\frac{a}{r}\right)^\alpha\right] - P(a)\left(\frac{a}{r}\right)^\alpha.$$

Determining $a\ddot{a}$ from the equation of motion (8) and converting to dimensionless variables, we obtain for the radial stresses

$$\Sigma_r = \left\{\frac{2\xi}{(\alpha-4)(1-\xi)}\left[\left(\frac{x}{z}\right)^\alpha - \left(\frac{x}{z}\right)^4\right] - \frac{\xi}{(1-\xi)(\alpha-1)}\left[\frac{x}{z} - \left(\frac{x}{z}\right)^\alpha\right]\left[\frac{(1-\xi)(\alpha-1)\varepsilon^{\alpha-4}}{\xi(\varepsilon^{\alpha-1}-1)} - \frac{2(\alpha-1)(\varepsilon^{\alpha-4}-1)}{(\alpha-4)(\varepsilon^{\alpha-1}-1)}\right]\right\}\left(\frac{dx}{d\tau}\right)^2 + \left\{\left(\frac{x}{z}\right)^3 - \left(\frac{x}{z}\right)^\alpha - \frac{\varepsilon^{\alpha-3}-1}{\varepsilon^{\alpha-1}-1}\left[\frac{x}{z} - \left(\frac{x}{z}\right)^\alpha\right]\right\}\frac{2\alpha\sqrt{\xi}}{3m(\alpha-3)\,Re} \times \frac{1}{x}\frac{dx}{d\tau} - \frac{P(x)}{P_0}\left\{\left(\frac{x}{z}\right)^\alpha - \frac{1}{\varepsilon^{\alpha-1}-1}\left[\frac{x}{z} - \left(\frac{x}{z}\right)^\alpha\right]\right\} - \frac{P_*}{P_0}\frac{\varepsilon^\alpha}{\varepsilon^{\alpha-1}-1}\left[\frac{x}{z} - \left(\frac{x}{z}\right)^\alpha\right] + \frac{k}{3mP_0}\left\{1 - \left(\frac{x}{z}\right)^\alpha - \frac{\varepsilon^\alpha - 1}{\varepsilon^{\alpha-1}-1}\left[\frac{x}{z} - \left(\frac{x}{z}\right)^\alpha\right]\right\}. \tag{12}$$

We determine the tangential component of the stress tensor σ_θ from the condition of plasticity (5):

$$\sigma_\theta = \frac{1-m}{1+2m}\sigma_r + \frac{k}{1+2m} + \frac{2\mu}{1+2m}\frac{u}{r}$$

or, in dimensionless form,

$$\Sigma_\theta = \frac{1-m}{1+2m}\Sigma_r + \frac{k}{(1+2m)P_0} + \frac{2\sqrt{\xi}}{(1+2m)\,Re}\frac{x^2}{z^3}\frac{dx}{d\tau}, \tag{13}$$

where $z = r/a_0$.

It can be concluded from the form of Σ_θ, that with a sufficiently high velocity of the cavity boundary, the appearance of tensile stresses is possible (it should be borne in mind that Σ_r in absolute magnitude is always negative). Calculations confirm this conclusion. Curves are shown in Fig. 4, illustrating the nature of the change of the stressed state of sand as a result of the explosion of a charge of $\simeq 27$ g ($\mu = 3 \cdot 10^5$ p). Curve 1 corresponds to a cavity radius of $a = 1.13\ a_0$, curve 2 – $1.17\ a_0$, and curve 3 – $1.33\ a_0$. The curves for Σ_r are not drawn; the radial stress during the entire motion will be compressive and will decrease monotonically from the cavity boundary to the shock-wave front. Already when $a = 1.17\ a_0$, a zone of intensive tensile stresses will extend from the boundary of the camouflet into the depths of the ground. As the stability of sand to an explosion is insignificant, we should put $\Sigma_\theta = 0$ in this region, by which the zone of cracking is introduced into the solution. This is not taken into account in the solution, as it still remains unclear how to describe the generalized state of the medium behind the front of the zone of cracking. The appearance of fissures was observed by the authors of [6] when carrying out experiments in sand with charges close in scale, with an increase of the cavity radius by a factor of 2.1 to 2.2. The difference between our value and the experimental value can be explained, on the one hand, by the imperfection of our theoretical model but, on the other hand, it is not at all obvious that the appearance of tensile stresses is accompanied rapidly by blurring of the image of the gas-cavity boundary, observed in the experiments.

It follows from the form of the expression for Σ_θ (13) that in a medium with zero viscosity the appearance of tensile stresses is possible only when the radial stress becomes comparable with the coupling forces in the ground, $\sigma_r \sim k$. For comparison, Fig. 5 shows curves of $\Sigma_\theta(z)$ for the same values of a as in Fig. 4. With experimental values of $k \sim 1$ kg/cm^2, this is possible only in the final stage of motion, close to the instant of cessation, and the magnitude of the tensile tangential stress in this case is insignificant – of order k. Hence, it follows that the appearance of tensile stresses at the initial stage of motion is due entirely to the viscosity of the ground.

It follows also from formulas (12) and (13) that the stress field around the cavity depends on the scale of the explosion. For large-scale explosions, the appearance of tensile tangential stresses must occur at relatively large cavity radii. Thus, on a sufficiently large scale of the effect, the ground around the cavity will be in a state of hydrostatic compression during the entire time of motion, and the appearance of radial fissures should not be observed. Moreover, for media with small internal friction (clay, saturated earth), the possibility of the appearance of tensile Σ_θ is less probable than for media with large values of m (loess, loam). This, in particular, explains the fact that during explosions in clay, comparatively smaller fissuring of the camouflet walls is observed than for loess or loam.

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